Boundless but Bundled: Modelling Quasi-infinite Dimensions in Ideological Space

Philip Warncke*

Flavio Azevedo[†]

March 01, 2024

Abstract

Ideological scales, derived from policy position items, are prevalent in political psychology and behavioral research. However, past scholarly practice shows little to no consensus as to how many such scales (i.e, ideological dimensions) researchers should consider in order to adequately capture the main political dividing lines among the mass public. A comprehensive literature review suggests, while a subsequent statistical simulation analysis confirms that the optimal number of latent ideological dimensions increases without bound as researchers include additional issue position items in their models. At the same time, nearly all ideological factors detectable within policy position data are sizably and positively correlated with one another. In light of these findings, a Bayesian hierarchical latent variable modeling framework is proposed which seeks to reconcile these conflicting qualities. The proposed model estimates ideology as a higher-level expression of correlated, lower-level building blocks. This model can evaluate whether particular socio-demographic or psychological predictors, such as income, gender, or egalitarianism, are consistently related to specific ideological sub-dimensions (e.g., economic, socio-cultural, racial ideology) or, instead, a generalized, unidimensional representation thereof. The present results underscore the potential of this approach, offering insights into the unique characteristics of different ideological factors and their overarching parent dimension.

Key words: "Ideology, Dimensionality, Scales, Measurement, Public Opinion"

^{*}University of North Carolina at Chapel Hill, pwarncke@live.unc.edu

[†]Utrecht University, f.a.azevedo@uu.nl

"Reactions to specific situations, involving conservative-radical issues, may be highly uncorrelated with each other.[...] Probably factor analysis applied to a large number of items would reveal highly unrelated groups of items for measuring conservatism." – **Theodore Lentz, 1938**

1. Introduction

Can the policy preferences of the mass public be captured by abstract ideological dimensions, and, if so, how many such dimensions are needed? For the better part of the past century, the question of ideological dimensionality has fascinated political theorists (Schattschneider 1960; Bobbio 1994; Freeden 2008; Maynard and Mildenberger 2018), social psychologists (Lentz 1938; Eysenick 1946; Rokeach 1973; Sibley and Duckitt 2008; Azevedo et.al. 2019), and public opinion researchers (Free and Cantril, 1956; Converse 1964; Stimson 1975; Inglehart 1997; Hochschild 2001; Ellis and Stimson 2012; Marble and Tyler 2022) alike. In broad terms, statistical models of mass ideology fall into unidimensional and multidimensional varieties. Proponents of unidimensional models posit that policy preferences can, in large part, be succinctly represented by a singular, generalized left-right or liberal-conservative spectrum (Downs 1957; Rabinowitz and McDonald 1989; Zaller 1992; Jost et.al. 2009; Ellis and Stimson 2012; Lauderdale et.al. 2018; Hare 2022). Conversely, advocates for multidimensional perspectives contend that a solitary left-right dichotomy falls short of capturing the complex nuances within mass political preferences (Inglehart 1997; Sibley and Duckitt 2008; Treier and Hillygus 2009; Feldman and Johnston 2014; Malka et.al. 2019; Atkinson et.al. 2021). According to this viewpoint, at least two substantially distinct ideological factors — such as those differentiating preferences for socio-cultural tolerance and economic redistribution — offer a more accurate representation of mass preferences.

Considerable academic debate surrounds the dimensional "essence" of ideology as a socio-psychological phenomenon (see Maynard & Mildenberger, 2018 for an overview), yet many applied researchers focus on a more pragmatic aspect of ideological dimensionality: They simply want to find the most parsimonious representation of a given set of political preferences. Lower-dimensional representations of policy position data have many desirable properties: they are easier to understand, simplify algebraic expressions, and are less likely to lead researchers into over-extrapolating information beyond the immediate sample. Nonetheless, overly reducing dimensionality can lead to significant information loss, resulting in unduly constrained models that may overlook vital insights only discernible in higher-dimensional analyses. Expressed more formally, given a set of k issue positions items, applied researchers often want to find the "optimal" number of latent dimensions $d \leq k$ which reduce the complexity of origi-

nal, k-dimensional space without discarding any substantively meaningful information. Researchers, in other words, want to distill as much statistical signal among a given set of items k into as few as possible dimensions, d, while discarding as much noise as possible in the process.¹

Focusing on policy ideology, the present manuscript shows that "optimal" choices for d likely only exist for fixed sets of k. In particular, extensive sensitivity simulations using graph-based, machine-learning dimensionality detection algorithms on policy position data from the American National Election Studies (ANES) and other data sources suggest that the most parsimonious choice for ideological dimensionality is by and large driven by the number of policy position items researchers have at their disposal. Put simply, dimensionality grows without bound as researchers select more and more issue items measuring ideology. This finding stands in stark contrast to psychometrically "better behaved" constructs like personality, for which the optimal number of latent dimensions converges rather quickly to 5-6 dimensions, no matter how many additional survey items researchers select for analysis.

At the same time, virtually all latent ideological dimensions detectable in large, nationally representative policy position data of the US public are positively and sizably correlated with one another. While distinct enough to warrant separate spatial representation, consistently positive inter-factor correlations indicate fundamental similarities shared across all latent dimensions. As a data-generating process, political ideology thus somewhat strangely exhibits both uni- and multidimensional properties: while resembling a bottomless barrel in terms of producing additional latent dimensions as a function of raw data input, it also acts like a large funnel, partially binding all such dimensions together.

In light of these findings, we propose an alternative modeling framework that aims to harmonize the uni-dimensional and multi-dimensional aspects of mass ideology. In particular, we outline a Bayesian hierarchical latent variable model which allows researchers to jointly estimate ideology as a higher-level expression of lower-level, multi-dimensional building blocks. This procedure is innovative insofar as it allows for effect estimation of external covariates *simultaneously* on multi- and unidimensional ideology. Researchers can, for instance, determine if a given socio-demographic or psychological predictor, such as income, gender, or egalitarianism, is consistently related to a particular ideological dimension such as economic, socio-cultural, or racial ideology, controlling for the impact of generalized, uni-dimensional ideology. We present results of such an analysis using ANES data to showcase the potential merit of this approach.

¹See Bennet (1969) and Trunk (1974) for more formal treatise of "optimal" latent dimensional dimensionality from an information theoretic perspective.

The remainder of this manuscript is organized as follows: the next two sections introduce the core concerns about ideological dimensionality and provide a motivating example for estimation issues that arise when researchers specify different models for the ideological space. In Section 4, the results of a systematic literature analysis covering six decades of American public opinion research reveal largely heterodox practices both in the number of issue items and latent dimensions researchers have employed in modeling policy ideology. A series of graph-based machine learning simulations in Section 5 statistically confirms a key pattern found within the empirical literature: the optimal choice for latent ideological dimensionality grows without bound as researchers select additional policy position items for analysis. At the same time, nearly all thus obtained dimensions are positively and appreciably correlated with one another. In light of these findings, we propose an alternative modeling strategy based on Bayesian hierarchical factors in Section 6, which seeks to unify the traditional distinction between multi- and uni-dimensional modeling approaches. A short discussion of the merit of understanding mass ideology as a hybrid construct, exhibiting both unidimensional and multi-dimensional qualities concludes this manuscript.

2. Ideology, Latent Space, and Dimensions

When referring to *ideology*, political scientists often conceptually evoke issue-based disagreements along a given spatial dimension. Whenever we label an actor, attitude, or issue position as "left-wing," "right-wing," "liberal," or "conservative," we implicitly state that this actor, attitude, or issue position can be located on a spatial axis together with a variety of other political objects that also relate to the same logic of (dis-)agreement (Downs, 1957; Converse, 1964; Ingelhart and Klingeman, 1976). As analytic tools, ideological dimensions can help organize political actors, issues, and opinions according to how similar they are to others based on a broader selection of substantially related political considerations. In other words, by labeling politicians such as Alexandria Ocasio-Cortez as left-wing and Ron DeSantis as right-wing we implicitly evoke an abstract spacial direction along which we can locate each of these actors based on their overall orientation towards a diverse set of political issues.

Public opinion researchers frequently apply the same logic to model the political issue attitudes held by ordinary citizens; survey respondents who are, for instance, in favor of immigration controls, abortion restrictions, and privatized prisons are typically mapped to one end, and adherents to open borders, access to abortion, and state-run prisons are mapped to the other end of the ideological space. Ideological dimensions can thus lend meaning to collections of attitudes citizens may endorse (Converse 1964; Achen 1975; Ansolabehere et.al., 2008).

A long-standing argument in American Politics holds that while political elites and party activists possess ideologically structured policy preferences (Poole and Rosenthal 1985; Converse 1986; Layman and Carsey, 2002; McCarty et.al. 2006; Ansolabehere et.al., 2008; Layman et.al., 2010; Hetherington and Rudolph, 2015), the same cannot be said of the general public (Converse 1964; Kinder and Kalmoe 2017; Baldassari and Gelman 2008; Fiorina et.al., 2011). Kalmoe (2020), for instance, concludes that "[p]olitical ideology is only polar, coherent, durable, and potent for a sophisticated minority – perhaps 20 – 30%." However, assessments like these ultimately hinge at least in part on assumptions about ideological dimensionality. In this case, the prominent ideological innocence thesis rests on the premise that mass ideology follows the same functional form as elite ideology – i.e. along a single left-right axis (Converse 1964, p. 12). If mass ideology is instead organized differently, for instance by operating on more than one salient dimension, a citizenry that is highly constrained along multiple dimensions might falsely appear as ideologically innocent if ideological thinking is only assessed along a single dimension (c.f. Treier and Hylligus, 2009; Carmines, et.al. 2011).

Consider a hypothetical libertarian – someone who holds liberal attitudes on a social issue dimension (e.g. being pro-gay marriage and pro-choice) yet harbors firmly conservative views on an economic policy dimension (e.g. being against Obamacare and state unemployment benefits). This individual will falsely appear as non-ideological on a uni-dimensional, left-right axis simply because their preferred issue configuration does not align with the unified belief system that is most commonly presented by political elites. The next section provides a more principled introduction using data from the 2016 wave of the ANES to illustrate some core concerns about the dependency between research findings and assumptions about the dimensionality of the ideological space.

3. Why Dimensionality Matters: A Motivating Example

Any mapping of multiple political issue preferences on a latent space requires assumptions about dimensionality. While this premise is an explicit feature of many latent variable models (e.g., Spearman 1907; Bollen 1980; Rash 1980), it is true even of the most elementary measurement instruments such as simple additive scales tallying the number of liberal and conservative positions a citizen endorses on a given set of political issues (e.g. Zaller 1992, p. 23f.). In fact, when constructing such an additive scale, researchers implicitly assume that all issue items are uniformly related to a single underlying dimension (which is measured without error). Additive scales thus represent the simplest possible, unidimensional model of mass ideology.

Perhaps less obviously, researchers also rely on an implicit model of latent dimensionality when mapping citizen's issue preferences onto a simple, orthogonal coordinate system, using a separate axis for each issue item. Such a modelling approach corresponds to an extreme version of the mulitidimensional perspective, in which each item warrants a separate spacial dimension.² In a strictly multidimensional model, the dimensionality of the joint attitude space is equivalent to the raw data space because each item is believed to represent a unique, non-overlapping latent construct.

Figure 1 depicts these contrasting approaches using three issue items taken from the 2016 round of the American National Election Studies (ANES): Abortion restrictions, government spending & services, and the size of the military budget. Panel A on the top-left, locates each Clinton (blue) and Trump (red) voter's position along a unidimensional scale formed by summing their responses of the three items.³ Panel B, on the other hand, follows a strictly multidimensional model by mapping each voter's responses on the same issues along three separate, orthogonal dimensions.

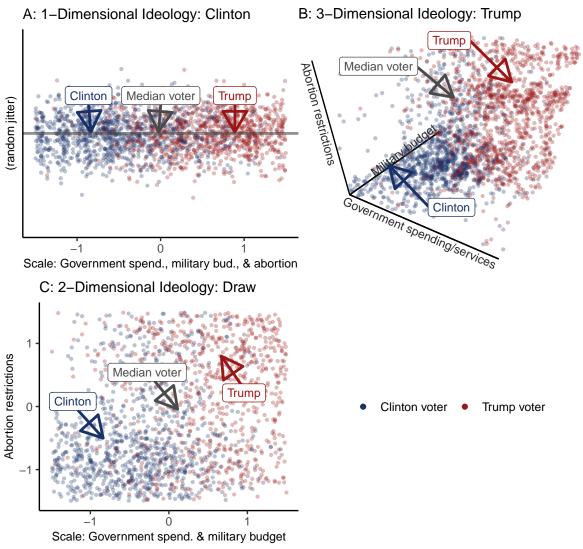
Why does dimensionality matter? Imagine you are tasked with finding out who among the main presidential candidates in 2016 was positioned closer to the ideological center of the electorate. In addition to each respondents' location within the joint ideological space, Figure 1 also marks the average of all respondents' perceived position of Hillary Clinton and Donald Trump on the same set of issues. Knowing these positions, one can simply measure the relative Euclidean distance of each candidate to the median voter position — i.e. the location of the voter who has as many voters to the right as to the left of herself.⁴ In the unidimensional model in Panel A, Hillary Clinton ekes out a narrow win as her position appears approximately 0.1 standard-deviation units closer to the ideological center. Panel B, however, suggests a much different outcome. According to the multidimensional perspective, Donald Trump scores an easy victory with a position 0.36 standard deviation units closer to the ideological center.

²See Ellis and Stimson 2012, p. 6f. for a discussion of this model.

³Each item were standard normalized and redirected such that negative scores represent liberal and positive scores represent conservative positions. The X axis represents the each voter's location on the axis. Panel A also features a small amount of vertical and horizontal jitter which was solely added for visualization purposes.

⁴In Panel A, this position is equivalent to the median score on the unidirectional scale. In Panel B's multidimensional representation, the median voter is located at the joint midpoint of the x, y, and z axis. Likewise, this median voter is surrounded by the exact number of voters in all spatial directions.

Figure 1: Who was Closer to the Median Voter in 2016?



Source: ANES 2016

In a nutshell, Figure 1 reveals that the same data, taken from the same respondents, can yield vastly different results simply because of different specifications for the number of dimensions in ideological space. How can this be? Simply put, different models of ideological dimensionality imply different weighting and aggregation rules for same underlying information. Since the unidimensional model treats each item response as if it was derived from the same latent construct, it implicitly up-weighs responses that conform with a single underlying liberal-conservative division. The additive model, in simpler terms, pushes respondents who consistently provide liberal or conservative answers out to either extreme along a single line. Respondents with mixed considerations, meanwhile, become "squeezed" towards the ideological midpoint — their original spacial deviations are implictly down-weighed within the joint, lower-dimensional space.

The multidimensional model, by contrast, employs uniform weights throughout because it treats no positional information as redundant. In this model, individual deviations in answering behavior possess the same interpretation in all spacial directions; a respondent who provides two liberal and one conservative response appears just as far away from the ideological center as a respondent who gave three consistently liberal or conservative answers. Because a strictly multidimensional model does not distinguish between cross-pressured and aligned individuals, it implicitly down-weighs responses that confirm with a general ideological dimension – should one exist – in favor of those respondents who do not conform to this dimension. As the above example shows, distortions resulting from different weighing and aggregation procedures imposed by different models of dimensionality can be substantial enough to determine the outcome of empirical research questions.

Although the above examples represent somewhat stylized versions of the unidimensional and multidimensional modeling approaches, any intermediate or mixed types of both suffers from the same inherent ambiguity. One such hybrid model, depicted in Panel C of Figure 1, combines the government spending/services and the military budget questions into a unidimensional scale while retaining abortion restrictions as a second, orthogonal dimension. By happenstance, this model results in an almost deadeven draw in terms of both candidates distance to the median voter (relative distance < 0.02 standard deviations).

More importantly though, neither solution depicted in Figure 1 is ostensibly superior to the others; while lower-dimensional models condense more information than higher-dimensional alternatives, it is not obvious how much, if any, of this information should ideally be condensed. In principle, researchers can rely on a variety of selection criteria that help them better adjudicate between different specifications for latent dimensionality. They could, for example, endorse a particular model if it, more so than the given alternatives, generates empirical estimates that are more likely to generalize beyond the immediate respondent or item sample. In the next section, we express this motivation more formally and discuss the usage of data-driven techniques designed to aid researchers selecting between different models of ideological dimensionality.

4. Data-driven Dimensionality Estimation and Practice in Political Science

Researchers are, in principle, at liberty of collapsing any number among k total items into any number of $0 < d \le k$ dimensions. Assuming one is restricted to forming simple additive scales as in the examples in Figure 1, the number of unique models allocating different sets of items among different ideological

dimensions, M, is given by:

$$M(k) = \sum_{d=1}^{k} S(k, d)$$

where S(k,d) is the Stirling number of the second kind, representing the number of ways to partition a set of k objects into d non-empty subsets. For k=3 items, there are 5 unique options at distributing these items along a maximum of $d_k=3$ separate dimensions. As summations over Stirling numbers grow faster than exponentially, the set of k=7 policy position items available on the 2016 ANES (providing data on both individual and candidate positions) already results in M=877 unique allocations. At k=14 model-based dimensionality configurations reach nearly 191 million.⁵

This mathematical reality has long prompted political scientists to employ data-driven strategies towards employing data-driven strategies that help adjudicate between different models of latent dimensionality (e.g. Stimson 1975; Miller and Miller 1976). This work by and large builds on earlier scholarship in quantitative psychology seeking to understand the dimensionality of fundamental psychological phenomena (e.g. Thurstone 1934); political scientists have started using psychometric tools to estimate latent dimensions in policy position data soon after these were proposed in that field (Kaiser 1960; Horn 1965; Cattell 1966).

At their core, all data-driven dimensionality algorithms seek to optimize $d \leq k$ such that the particular set of latent factors, d_k , likely generalizes beyond the immediate sample; too many factors risk over-fitting the data and afford too much weight to idiosyncratic characteristics of the immediate sample. Conversely, too few latent factors likely under-fit the data, potentially obscuring or muffling important features of the underlying data-generating process. To paraphrase Einstein, finding the optimal number of latent dimensions amounts to making the world "as simple as possible, but no simpler".⁶

Reckase (1990) offered an influential definition of dimensionality in statistical terms, referring to it "as the minimum number of mathematical variables needed to summarize a matrix of response data." This definition points to a critical aspect of dimension reduction: as no model relying on fewer variables than the original sample can provide a perfect reproduction thereof, researchers need to establish benchmarks about what qualifies as an adequate summary of the sample characteristics.

⁵Moreover, any modeling strategy allowing for weighted sums and/or arbitrarily correlated latent dimensions trivially results in infinitely many unique model specifications.

 $^{^6}$ See Robinson, A. (2018) Did Einstein really say that?, Nature News. Available at: https://www.nature.com/articles/d415 86-018-05004-4#:~:text=%E2%80%9CEverything%20should%20be%20made%20as,possible%20without%20having%20t o%20surrender

Unfortunately, no universally accepted benchmarks exist which jointly optimize for simplicity and information retention. In practice, scholarly communities have relied on a variety of threshold statistics, including those derived from correlation matrix decomposition (Kaiser 1960; Horn 1965; Garrido et.al. 2013), likelihood-based indices in factor models (Hu and Bentler 1999; Chen et.al. 2008), information criteria extracted from item response theory models (Treier and Hylligus 2009), posterior dimension distributions estimated as part of Bayesian factor analysis (Conti et.al. 2014), and information entropy fit measures of regularized correlation networks (Golino et.al. 2021).

To what extend are statistical dimensionality estimation techniques used in past research relying on ideological summary scales? Figure 2 summarizes the results of a systematic literature analysis on ideological scale construction covering six decades of applied American political behavior and political psychology research.

In order to delineate a literature sample for this analysis, a list of keywords including "ideological summary scale[s]", "political position scale[s]", "political value scale[s]", "political issue[s]", "political preference[s]", "policy ideology," "operational ideology", and "ideological dimensionality" was supplied to the ProQuest database of academic publications (search conducted in March 2022). Only texts that were published in peer-reviewed journals or books compiled by academic publishers were retained in the sample. In addition, each manuscript must meet all of the following criteria:

- 1. it must conduct quantitative analysis on at least one probability sample of US adults (opt-in and convenience samples are excluded)
- it must use at least one summary scale formed of at least two policy position or political preference items, i.e. items asking participants to normatively evaluate a particular function or role of government or to select between different policy options.
- 3. each ideology scale can include no more than one non-policy position item (usually symbolic ideological identities or feelings thermometer scores of salient political groups)

These inclusion criteria yielded a sample of 74 publications. Although no deliberate selection based on academic disciplines was undertaken, the final sample is heavily dominated by publications in political science (81%), followed by political and social psychology (8%), sociology (7%), and economics (4%). Alongside meta-information such as publication outlet, date, and survey sample, we recorded the number of issue items used to build ideology scales, how many such scales (i.e. ideological dimensions) the authors constructed from these, and whether they reported to have used any statistical technique

designed to infer latent dimensionality from raw data. We broadly define such techniques to include eigen-decomposition based strategies (e.g. scree-plots as part of principle components analysis), model fit comparisons between factor models of different dimensionality, and comparisons of information criteria in classical test- and item response theory models (e.g. improvements in AIC or BIC statistics).

We found 122 distinct operational ideology scales across all publications; this number exceeds the publication sample because numerous manuscripts contain more than one unique scale, often fitted to different data in a different part of the analysis. A plurality of scales feature 10 or less items but there is a substantial degree of heterogeneity in this regard $(SD_k=9.4)$; a few studies even include scales constructed from 40 or more unique items. Practices for choosing the optimal number of ideological dimensions are similarly heterogeneous: While most studies invoke uni-dimensional scales, more than half of models (51%) feature scales along more than one dimension.

In terms of statistical dimensionality detection, only about one quarter of articles (26%) report the results of any data-driven analysis designed to detect the optimal number of latent dimensions. The literature sample shows little to no evidence that usage of such techniques is related to publication year r = -0.17; SE[r] = 0.12 and the number of selected issue items (r = -0.02; SE[r] = 0.09), and only a very very weakly correlated with the number of dimensions used in the study (r = 0.24; SE[r] = 0.09).

5 4 Ideological dimensions d 3 2 Pearson r = 0.51 3 . 12 15 18 21 . 24 27 30 . 33 36 39 42 9 Issue items k

Figure 2: Issue Items and Latent Dimensionality among Literature Sample

Author(s) reported using latent dimensionality detection technique: • no • yes

Note: Sample consists of 124 models found within 72 texts. Line fitted with LOESS algorithm (span = 0.75)

The perhaps most concerning discovery, however, is depicted in Figure 2: among the published literature, the number of selected issue items k correlates strongly and positively with the number of ideological dimensions d used in the same analysis ($r_{d,k}=0.51; SE[r_{d,k}]=0.08$). Picking a set of items, in other words, appears to be strongly informative of how many scales researchers construct from these. This should be reason for worry as a strong dependency between input data and "optimal" dimensionality does not help solve, but worsen the key concerns about estimate variability resulting from alternative model specifications such in the examples outlined in Section 3. Put simply, data-driven procedures optimizing for latent dimensionality (i.e., finding the best solution for d given k) should not themselves strongly depend on the very model input specifications they are designed to optimize over (i.e., if the "best" k is itself is determined by d). In this case, optimal solutions for latent dimensionality only exist for a fixed set of input data and are not likely to generalize beyond the immediate data researchers have at hand. To more fully evaluate this predicament, the next section investigates the statistical relationship between optimal latent dimensionality and the number of political issue items using large-n simulations based on ANES data.

5. Dimensionality Simulations

An ideal data source for the purpose of investigating the association between number of survey items and latent dimensionality possesses a large number of unique policy position items, k, administered to a large, nationally representative sample. The 2012 wave of the ANES features an unprecedented number of $k_{max}=74$ unique policy position questions, making it an ideal for a comprehensive statistical simulation. Appendices B and C replicate the same procedure for the ANES waves of 2000 and 2020, at slightly smaller issue item pools ($k_{max}=42,62$), respectively. Appendix D repeats the same analysis for a non-ANES data source, using the 2018 wave of Cooperative Congressional Election Studies.

For each of these issue item samples, we simulated $\approx 10,000$ latent ideology models with item sets ranging from k=3 to $k_{max}-3$, using iterative process of item selection, dimensionality estimation, and scale construction. Each simulation iteration proceeds in three steps: First, a random sample from the population of unique policy position items of size k is drawn. We next estimate a polychoric item-correlation matrix from this sample. A latent dimensionality detection algorithm subsequently suggest the number of latent dimensions, d, alongside a best-fitting item loading pattern. Finally, a confirmatory factor model is fit to the raw data to check for model convergence and to extract the fitted inter-factor correlation pattern. Since estimating the optimal number of latent dimensions is by far the most complex aspect of the simulation, we describe this part in more detail below.

Across all simulations, we rely on Exploratory Graph Analysis (EGA) – a graph-based, two-stage machine-learning procedure proposed by Gollino et.al. (2021) – to find the most parsimonious solution for latent dimensionality (see also Gollino and Epskamp et.al. 2017). In the first stage, EGA finds the optimally sparse¹⁰ representation of the item correlation matrix using Gaussian Least-Absolute-Shrinkage (G-LASSO). In a second stage, the sparse correlation matrix is passed to the Leiden community detection algorithm (Traag et.al. 2019) with a varying set of sensitivity input parameters.¹¹ By allowing random variations within the sensitivity parameter space, the algorithm is able to identify the network community solutions associated with the lowest total information entropy.¹² The thus obtained number

⁷Items qualify as long as they meet Ellis and Stimson's (2012, p. 16) definition of operational ideology, that is items probing for respondents position on "the proper role and scope of government action and values". A full list of items taken from the 2012 ANES appears in Appendix A.

⁸The 22 items selected from the 2018 CCES are identical to those used in Fowler, A., Hill, S.J., Lewis, J.B., Tausanovitch, C., Vavreck, L. and Warshaw, C., 2023. Moderates. *American Political Science Review*, 117(2), pp.643-660. Appendix D lists the selected items.

⁹Less than 5% of models failed to converge.

¹⁰Optimum sparsity is obtained by searching a field of 1,000 candidate lambda parameters and selecting the lambda value associated with the minimum model-generated Extended Bayesian Information criterium (EBIC).

¹¹We use 1 through 10 walkrap cut-points. See section "EGA.fit" of the EGA_net R-package manual (p. 36). (https://cran.r-project.org/web/packages/EGAnet/EGAnet.pdf)

¹²Further details on this method can be obtained in Gollino et.al., 2021.

of item communities is conceptually equivalent to the optimal number of latent dimensions governing the data-generating process. As an added benefit, membership in EGA communities propose optimal loading patterns which can be used to fit confirmatory factor models (Gollino et.al. 2021).¹³

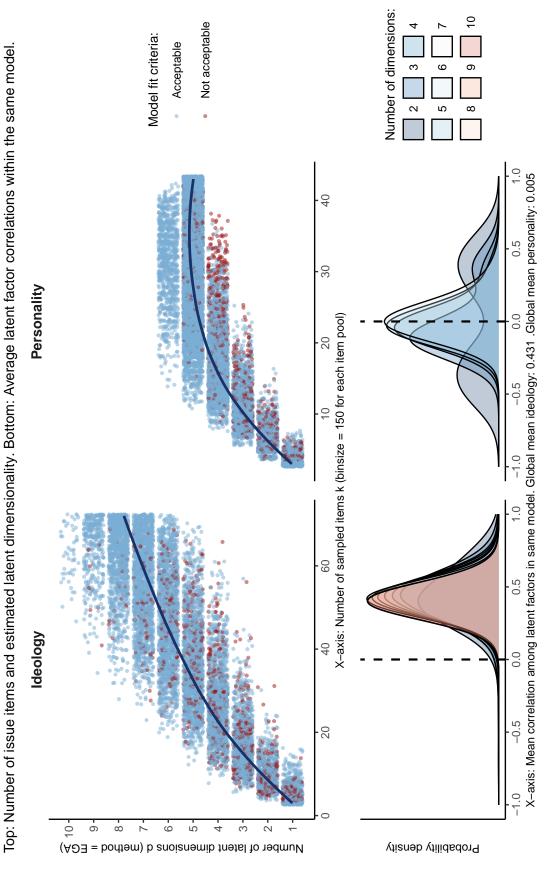
The results in the upper left-hand panel of Figure 3 strongly confirm the key pattern found within the published literature: larger item buckets require additional ideological dimensions to adequately summarize the underlying attitude space ($r_{d,k}=0.87; SE[r_{d,k}]<0.01$). Ideological dimensionality, in other words, grows without bound as more data is used to estimate it. Appendices B through D reveal very similar trends across comparable data sources. Appendix E confirms the same result for the 2012 ANES using Horn's parallel analysis (1965) as an alternative, yet comparably precise (Gollino et.al. 2021) dimensionality detection method. Figure 3 additionally visualizes CFA fit for each model to assess whether commonly used fit criteria can be relied used to systematically distinguish between dimensionally underand over-fitted models across the range of k. Using a reasonably strict fit criteria battery, ¹⁴ almost eight in ten randomly generated models fit the data extremely well. More importantly, the share of ill-fitting models above the trend-line is almost identical to the corresponding share below the trend-line (0.48 to 0.52), which further suggests that "ideal" dimensionality grows with the number of issue items.

To what extend might boundless dimensional growth simply result from population heterogeneity in terms of political sophistication or ethno-racial diversity? If unidimensionality is indeed a fundamental feature of mass ideology in the United States (e.g. Ellis and Stimson, 2012), such a feature may not be as easily detectable in nationally representative respondent samples which inevitably include many politically disinterested and unsopisticated individuals (Converse, 1964; Kinder and Kalmoe, 2017). Furthermore, different ethnic and racial groups might understand ideology differently, which could obscure an otherwise well-structured data generation process. We specified a series of sub-demographic analysis to test if boundless dimensional growth is unique to low education (Appendix F), low political knowledge (G), white/non-white (H) populations using the 2000 and 2012 waves of the ANES. The results strongly suggest that ideological dimensionality grows without bound across all educational and political knowledge quartiles, as well as white, and minority-only sub-samples.

¹³Replications using Horn's (1965) parallel analysis reveal very similar patterns of boundless dimensional growth.

¹⁴The joint set of criteria include CFI, TLI > 0.9, RMESEA lower 90th confidence percentile < 0.08, SRMR < 0.08. A model fits well only if it jointly meets all of these criteria.

Figure 3: Item Samples and Latent Dimensionality in Policy Ideology and Personality Data



Some readers may question if the higher-dimensional solutions in Figure 3 simply contain a large number of co-linear factors. If this was the case, additional dimensions would offer little to no additional informational value. Conversely, others might question whether the population of ANES policy position items was well-suited to capture operational ideology as as a single, underlying construct to begin with. If these items are only weakly or quasi randomly related, they likely yield quasi-random, substantially meaningless latent dimensions. In this case, one should expect the average factor correlation to be centered at zero and obtain equally many positively as negatively correlated factors.

Evidence for these methodological objections should manifest in divergent ways: if the latent factor solutions exhibited strong positive correlation all solutions encapsulate essentially identical information, or, if they exhibit no clear correlation tendency at all, the solutions would only capture highly idiosyncratic information. Intriguingly though, the bottom right panel in Figure 3 reveals that neither is the case: based on the probability density distributions of inter-factor correlations within each model with a minimum of two dimensions (which constitutes approximately 96% of the models), hardly any factor pair within the same model is negatively correlated; only a handful (< 2%) approach orthogonality. Simultaneously, only very few factor solutions are very highly correlated (less than 4% are larger than 0.75). Instead, moderate association pairs between any two latent factors dominate the multi-factor space.

Panel B further illustrates that the number of latent dimensions does not display any systematic relationship with the inter-factor correlation patterns. This means that as one expands the item sample and estimate additional dimensions, latent ideological factors do not simply become more similar to one another. Rather, the average factor correlation of $\bar{r}_{ideology}=0.43~(SE[\bar{r}]<0.01)$ suggests that the majority of solutions at least in part capture the essence of a single underlying conceptual family, albeit with a substantial degree of heterogeneity. In sum, the simulated latent factors appear dissimilar enough to warrant distinct dimensions of measured operational ideology. Nevertheless, they also demonstrate a sufficient degree of similarity, loosely linking them to the same class of constructs. In the last section, we will circle back to this finding, as it provides the basis for reconceptualizing mass ideology as a hybrid phenomenon, exhibiting both uni- and multidimensional characteristics.

Finally, one might object that boundless dimensional growth could simply be a feature – or rather, a flaw – of the dimensionality detection algorithms employed here. More specifically, these algorithms might be overly sensitive towards minute evidence for the existence of additional latent dimensions, especially for large item buckets k. To address this concern, one could ask how the unbound dimensional nature of political ideology stacks up against more well-established construct in quantitative psychology — such as

personality — as k increases? Using the identical simulation procedure, the right-hand panels in Figure 3 presents estimates for latent dimensionality and inter-factor correlation for a large, publicly available personality questionnaire featuring answers from 288 US undergraduate students on $k_{max}=44$ items. Although dimensionality estimates also initially grow rapidly here, they plateau at the widely accepted latent factor structure between 5 and 6 dimensions at $k \gtrsim 25$. Moreover, virtually all latent personality dimensions are nearly perfectly orthogonal to one another $\bar{r}_{personality}=0.005$ ($SE[\hat{r}=0.0018]$). 16

What consequences does boundless dimensional growth for policy ideology bode for researchers who rely on data-driven dimensionality detection algorithms in order to adjudicate between alternative models? Reconsider the opening problem about determining which of the main presidential candidates in 2016 was ideologically positioned closer to the median voter. One unsettling implication of the present finding is that the "optimal" model for latent dimensionality is very likely a direct function of the number of issue items researchers happen to feed into their analysis. Rather than determining more broadly applicable solutions, that is detecting models that produce estimates which likely generalize beyond the immediate item and respondent sample, the simulation analysis strongly suggest that global solutions for ideological dimensionality are impossible to obtain. In Figure 4, we confirm this predicament by plotting the results of a large class $n \approx 100,000$ models, each of which is calculating the perceived ideological distance between Clinton and Trump in 2016 across the sets of available political position items on both candidates

 $1 < k \le 7$, while randomly allocating each set of k along $1 < d \le 7$ dimensions. ¹⁷

¹⁵Data source: Ordinal Data of the Big Five Inventory (Luo, 2005). These data were collected as part of a the study on personality and relationship satisfaction. N = 228 undergraduate students at large, US-based public university. Used are all self-ratings on the 44 item Big Five Inventory proposed by John et.al. (1991).

 $^{^{16}}$ In Appendix X, we cross-validate the same findings for another publicly available personality data-set (Tunguz, 2018) at $k_{max}=50$. This data source is based on an a massive online convenience sample administered by Open-Source Psychometrics, featuring n > 1 million responses from >180 countries. See https://openpsychometrics.org/tests/IPIP-BFFM/ for the item battery.

¹⁷This simulation introduces additional variation across models by sampling a subset of 500 out of 2,700 complete-cases responses.

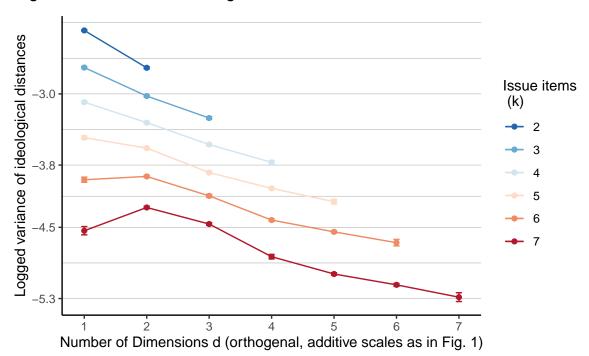


Figure 4: Variance of Ideological Distances across Model Dimensionalities

Source: ANES 2016. Y axis displays the variance of ideological distance estimates between the median voter a 2016 presidential candidates based on 1 million randomly generated models, each sampling 500 respondents and a pool of issue items k, randomly allocated along d orthogonal dimensions.

The y-axis in Figure 4 displays the variance of the thus obtained ideological distance estimates. This metric affords direct comparisons of the relative performance of each dimensionality configuration at fixed sets of items; "better-behaved" models tend to produce estimates that vary less for a given respondent and item pool. Figure 4 shows that models generally produce estimates with the lowest possible variance whenever the bucket sizes k exactly equals the number of latent dimensions, d. Once again, "optimal" dimensionality appears to grow without bound as researchers consider additional items in their models. Unfortunately for researchers, this implies that data-driven benchmarks are strongly influenced by the size of the employed item bucket and thus cannot provide grounds for selecting dimensionality configurations that likely generalize beyond the immediate sample data.

Pivoting back to the comparison between personality and ideology in Figure 3, this issue is nicely mirrored in direct comparison of the behavior of CFA-derived model fit statistics between ideology and personality. Among the personality models, fit metrics largely succeed in flagging dimensionally underfitted models, especially for item buckets $k \geq 20$; here, researchers would reject the lions' share of 4-dimensional solutions in favor of 5 or 6-dimensional alternatives. As stated above, model fit is all but randomly distributed across the range of k in the ideology simulation. For personality, but not ideology

¹⁸The share of well-fitting/ill-fitting $d \le 4$ at $k \ge 20$ is approximately 0.01 (p < 0.001) while nearly all models of $d \ge 5$ fit the data well (0.92 at p < 0.001).

ogy, this implies that researchers can generally expect that CFA fit systematically distinguishes between dimensionally over- and under-fitted models. In short, model fit criteria appear to perform well for dimensionally capped constructs like personality. They fail, however, in providing generalizable benchmarks for dimensionally unbound data.

The present simulation analyses, in a nutshell, show that ideological dimensionality grows without bound as researchers incorporate more information measuring it. Importantly, the same does not happen for psychometrically "better-behaved" concepts like personality; here, the number of survey items does not lead to a proportional increase beyond 5-6 latent dimensions. At this point, some readers may inject that policy position items, particularly those fielded as part of large political attitude surveys, are not likely to have been designed to reproduce a particular dimensional structure in the first place. Survey makers must carefully evaluate various considerations, including financial ones, when crafting questionnaires designed to capture the major political dividing lines among large, heterogeneous societies. Making sure that a given policy position battery neatly reproduces a particular model of latent dimensionality – as with personality questionnaires – is almost certainly not one of them. In this sense, direct comparisons between constructs like personality and policy ideology rightfully appear ill-posed. However, the critical point is that applied researchers frequently use policy position scales as if they were dimensionally bound. Practitioners, in other words, tend to treat ideology as if it was psychometrically as well-behaved as personality. As discussed in Sections 3 and 4, this practice unfortunately leads to double ambiguity in applied settings both because empirical estimates can fluctuate considerably under alternative dimensional specifications and researchers generally lack objective standards that could help adjudicate between them (e.g., on the basis of parsimony).

However, one critical silver lining should be reiterated: While personality is constrained by a fixed number of orthogonal factors, virtually all latent dimensions identified in policy position data are strongly and consistently positively correlated with one another. Although complex enough to warrant separate spatial representation, all latent ideological dimensions seem to be tethered to an overarching, yet somewhat imprecise, uni-dimensional origin. In the last section, we suggest a blueprint for how this common information can best be utilized to model mass ideology as a data-generating processes that may be dimensionally unbound, yet partially bundled together.

6. Unifying multi- and unidimensional perspectives in Bayesian hierarchical factor model

The preceding sections established how empirical models of political ideology can produce contrasting outcomes depending on the model of latent dimensionality that researchers select. They also revealed that past scholarship in political science largely tended to follow the trend evident in data-driven estimates for optimal latent dimensionality - ideological dimensions grow with the number of policy position items researchers include in their models. One consequence of the dimensionally unbound nature of mass ideology is that tools designed to detect "optimal" dimensionality likely suffer from strict limitations in terms of external validity. However, the simulation analysis also strongly suggests that the boundless multidimensional space possesses an inherent degree of structure; although separate ideological factors are warranted to account for the growing complexity as researchers supply more and more policy items, nearly all latent factors appear related through sizable, unidirectional correlations.

Neither the unidimensional nor multidimensional modeling approach can individually accommodate both of these aspects. Boundless dimensional growth might support the multidimensional perspectives yet strong resemblances among the collection of latent factors point to a joint, latent origin, forcing all factors into partial alignment. Furthermore, any multi-dimensional model of finite dimensionality is ultimately incomplete as researchers working with larger item sets will likely find evidence for better-fitting, higher-dimensional alternatives. We argue that a hybrid modeling approach which borrows elements from both modeling frameworks can better account for the shortcomings in either. Rather than strictly imposing a uni- or finite multi-dimensional model, the proposed framework treats mass ideology as a meta-concept which can manifest itself in a (potentially infinite) number of concrete, policy-specific dimensions which are loosely related to a general, albeit somewhat diffuse hyper-dimension.

Figure 5 outlines the blueprint for a Bayesian hierarchical factor model featuring a quasi-infinite number of area-specific sub-dimensions ("Dimension_x") explaining the covariance structure among a given set of policy position items (labelled "I"). This modeling strategy takes advantage of the quasi-hierarchical dependency between distinct ideological dimensions (such as social, economic, and/or racial ideology) and a generalized dimension which captures the common essence among the former. The model further assumes that all of these sub-dimensions originate from a single, albeit somewhat diffuse, hyperdimension (labeled "IDEOLOGY" in Figure 5). While the sub-dimension level features freely estimated loading structures (γ 's), the uni-dimensional hyper-factor is drawn from an unweighted average across all sub-dimensions (i.e., the loading coefficients are constrained to unity). We believe that this modeling approach has key advantages over conventional strategies as it can flexibly be extended to accommodate

any number of political position items and seamlessly integrated into dimensionality detection workflows such as the one presented in Section 5.

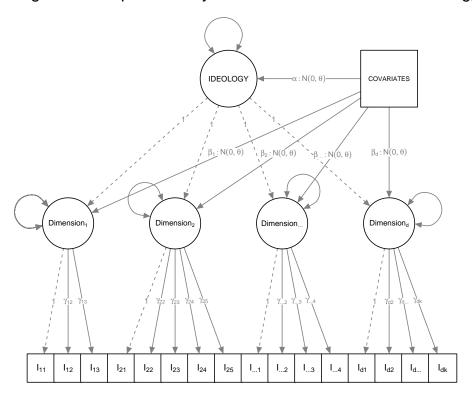


Figure 5: Blueprint for Bayesian Hierarchical Model of Ideology

Furthermore, the hierarchical ideology model features informative prior distributions on optional, external predictors, allowing for *simultaneous* effect estimations of such predictors on uni-dimensional (labeled α in Figure 5) and multi-dimensional (β 's) expressions of ideology. Effects of external predictors such as income or racial resentment can thus be decomposed into particularities of certain sub-dimensions (such as economic or racial ideology) and effects that generalize across all dimensions. In this way, the proposed framework can be interpreted as a causal mediation model that differentiates between direct effect of external predictors w.r. to particular sub-dimensions (β) and mediated effects (α) that first pass through the common hyper-factor. Note that this kind of decomposition is impossible with conventional maximum-likelihood methods as the set of lower-level factors is jointly co-linear with the uni-dimensional hyper-factor under the assumption of uniform prior distributions. Informative priors hence provide a critical asset of Bayesian hierarchical models as they aid in the empirical differentiation between the common, as well as the unique, properties of every sub-dimension detectable by data-driven dimensionality algorithms.

Figure 6 showcases one possible application of the proposed estimation strategy. Here, we present the results of a series of models featuring socio-demographic and psychological covariates which can help substantially interpret differences between ideological sub-dimensions and a unifying hyper-factor. As a data foundation, we rely on the 2000 ANES as this particular wave appears as the single most frequently used data source in the literature analysis and provides the basis for several, high-impact publications on the nature of ideology in the American public (Lupton, et.al., 2015; Feldman and Johnston, 2014; Treier and Hillygus 2009; Anasobelere et.al., 2008). From this dataset, we selected a set of k=32 policy position items. All of these items have been used at least once in ideological scale construction among a set of 9 relatively recent, high-quality publications; a number of well-established public opinion researchers, in other words, have found these particular items to be representative of the major political topics that divided the US electorate at this moment in time. Using EGA to detect latent dimensionality within this attitude set, we obtained d=6 optimally distinct, positively correlated ($\bar{r}=0.45; SE[\bar{r}]=0.16$) ideological sub-dimensions. Based on their respective item loading patterns, these dimensions could be labeled as 1) poverty reduction, 2) New Deal issues, 3) socio-cultural issues, 4) racial justice, 5) moral & sexual chauvinism, and 6) anti-immigrant chauvinism.

What substantial claims can be made about these sub-dimensions and their over-arching parent factor? Figure 6 presents the results of a series of models fitting a series of socio-demographic and psychological predictors²¹ to a joint model featuring the above-listed sub-dimensions and a single hyper-factor accounting for the common variance among these. The predictors broadly fall into two categories: the first, depicted in the top panel of Figure 6, includes political partisanship, ideological self-identification, and racial resentment. Generalized ideology – i.e. the shared essence among all ideological sub-dimensions – is strongly related to each of these predictors as seen by the sizable coefficients across the top row in Figure 6. Predictors in the first category, in other words, strongly and relatively homogeneously predict the common conceptual core captured across all basis dimensions. At the same time, some of the sub-dimensions show substantial relevance over and above generalized ideology. The model for liberal-conservative self-identification, for example, predicts substantive positions on gender issues to matter above and beyond respondents' general ideological orientations. Similarly, and perhaps not surprisingly, racial resentment pulls racial conservatism and anti-immigrant chauvinism further to the right based on

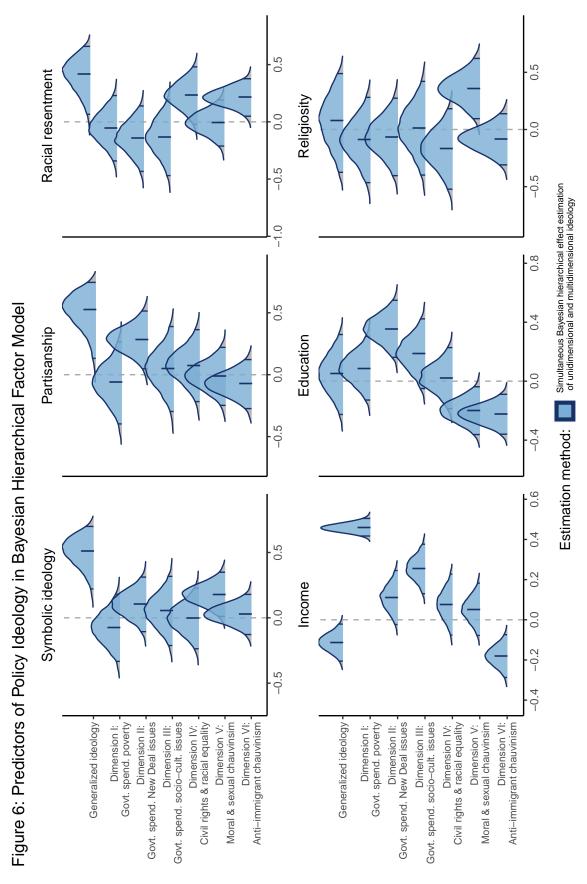
¹⁹These works are in alphabetical order: Ansolabehere et.al., (2006, 23 items), Barker and Tinnick (2006, 11 items), Carmines et.al. (2011, 21 items); Claggett et.al., (2014, 21 items); Feldman and Johnston (2014, 7 items); Layman and Carsey (2002, 17 items); Lupton et.al., (2015, 11 items); Malka et.al. (2014, 13 items); Treier and Hillygus (2009, 18 items). A list of all 32 items common to all papers appears in Appendix I.

²⁰Appendix H visualizes the loading structure for the 6+1 measurement model.

²¹These are the same predictors as in Feldman and Johnston's (2014) influential research article, *Understanding the determinants of political ideology: Implications of structural complexity.*

what would expect given respondents' generalized orientation across all ideological dimensions.

By contrast, predictors in the second category, which appear in the bottom panel of Figure 6, show no clear relationship with generalized ideology; this can be seen by the largely zero-centered coefficients across the top row in the bottom panel. These include covariates like income, education, political knowledge, and religiosity. However, each of these predictors shows unique associational patterns across the ideological sub-dimensions. Controlling for generalized left-right orientations, income, for instance, strongly predicts conservative stances on anti-poverty issues; anti-immigrant sentiment, meanwhile, moderately decreases with income. Other predictors, such as level of education, are positively related to ideological sub-dimensions dealing with economic redistribution, yet negatively predict sociocultural tolerance.



Source: ANES 2000. Bayesian plots feature the 2.5th, median, and 97.5th posterior percentiles values. Number of items: 32. Number of latent dimensions: 6.

In sum, by combining key features from unidimensional and multidimensional model approach, the present blueprint for modeling ideology as a Bayesian hierarchical factor not only offers more flexible solutions given particular choice options for item selection and latent dimensionality, but also allows for investigations that might contribute towards a more fine-grained understanding of the socio-demographic antecedents of mass ideology.

7. Conclusion

Empirical research on mass ideology is intimately linked to questions about dimensionality. When assessing the degree to which citizens adopt ideologically congruent beliefs, researchers typically evoke one or more ideological dimensions as yardsticks against which to evaluate how well citizen preferences match those pre-defined standards. When studying the prevalence of ideological polarization, researchers need to make assumptions about the dimensionality of ideology to get an idea about what ideological "poles" mean in the first place. When measuring the relative ideological distance between voters and candidates, we first need to establish ideological dimensions to scale the space in which we wish to measure distances. In short, we cannot evaluate the empirical claims about mass ideology without *a priori* pontificating about ideological dimensionality.

Despite their ubiquity in applied research, assumptions about ideological dimensionality are themselves rarely the sole subject scrutiny. In this manuscript, we show that ostensibly inconsequential choices about the number of issue items researchers select and the number of separate scales, i.e. latent dimensions to construct from these can have a profound impact on empirical findings. More fundamentally, the present analysis uncovered a deep connection between both aspects: Meta-data on research practice from 74 high-quality research publication in American political behavior shows that researchers who select larger issue item pools tend to model the ideological space in higher dimensions. A large-scale statistical simulation analysis including all available policy-position items within high-quality public opinion surveys subsequently confirmed that this tendency approximates a fundamental feature of the American attitude space: ideological dimensionality grows without bound as one increases the number of issue positions under consideration. At the same time, virtually all latent dimensions detectable within the mass policy preference space are positively and appreciably correlated with one another; separate ideological dimensions, in other words, at least partially capture the same underlying information. Mass ideology should therefore perhaps neither strictly be understood as a unidimensional nor multidimensional phenomenon. Instead, we argued that ideological preference more likely exists in a hybrid

state characterized by distinct yet related concepts, somewhat akin to the members of an extended family. To better differentiate the common conceptual nucleus from characteristics unique to only a subset of ideological sub-dimensions, we proposed an alternative modeling framework which allows researchers to model policy ideology simultaneously as a unidimensional or multidimensional construct.

We would like to conclude by returning to the initial question: how many dimensions are needed to faithfully capture the essence of citizens' policy preferences? Based on the present analysis, one might be inclined to say, both one and many. Such an answer, however, might appear unsatisfactory to readers who might wish for a more clear cut solution to one of the most perennial questions about the nature of mass ideology. At the same time, such clear-cut answers are already abound among the extant literature (see Maynard and Mildenberger 2018 for an overview). From a purely empirical perspective, there is clear merit to both the uni-dimensional and multi-dimensional perspectives. On the one hand side, there is undeniable evidence for multi-dimensionality among the most widely cited public opinion data sources on Americans' political issue positions. On the other hand, the same data can, in principle, give rise to quasi-infinite number of positively aligned dimensions which tells us that the multi-dimensional ideological space is ultimately bound to a single, if somewhat diffuse, parent dimension.

Our view likely best aligns with a small but growing body of literature that regards ideology as a family resemblance concept (e.g., Cochraine, 2015; Gidron 2020). Wittgenstein (1953), who first coined this term, argued that many social concepts, particularly those involving social cognition, embeddings, and applications, follow logic of relational resemblance: While a given pair of blood-related family members is likely to share some physical similarities such as similar shape of nose, eyes, or chin, it is nearly impossible for all family members to exhibit a single shared feature. Instead, it is the *collection of shared features* which differentiates one (conceptual) family from another. Our analysis shows that different ideological sub-factors such as racial, socio-cultural, or wealth-redistribution ideology exhibit numerous unique characteristics; few if any individuals are consistently liberal or conservative on all of these dimensions. Collectively however, the set of ideological factors share substantial enough similarities (in the form of consistently positive correlations) to classify them as belonging to the same conceptual family — even if the nucleus of that family remains slightly obscure under close scrutiny.

Finally, we think at least two important caveats appear in order to better put the present contribution into perspective. First, we exclusively focused the empirical part of the analysis on operational ideology – that is the summation of citizens' preferences on specific political issues and beliefs about the proper scope and function of government. Although this definition captures a large body of work on mass ideology,

particularly in political science, one should certainly acknowledge other perspectives on conceptualizing and measuring ideology. Future research could extend the present definitions and conceptualizations to as scale-models of psychological conservationism (Wilson and Patterson, 1968) or social dominance orientation (Sidenous and Pratto, 1999), for example. Secondly, we purposefully restricted the literature sample and statistical simulation analyses to data from the United States. Future work should cross-examine the present findings in different countries to better evaluate the core claims across different political and cultural contexts.

Literature

Achen, C.H., 1975. Mass political attitudes and the survey response. *American Political Science Review*, 69(4), pp.1218-1231.

Ansolabehere, S., Rodden, J. and Snyder, J.M., 2008. The strength of issues: Using multiple measures to gauge preference stability, ideological constraint, and issue voting. *American Political Science Review*, 102(2), pp.215-232.

Azevedo, F., Jost, J. T., Rothmund, T., & Sterling, J. (2019). Neoliberal ideology and the justification of inequality in capitalist societies: Why social and economic dimensions of ideology are intertwined. Journal of Social Issues, 75, 49–88.

Baldassarri, D. and Gelman, A., 2008. Partisans without constraint: Political polarization and trends in American public opinion. *American Journal of Sociology*, 114(2), pp.408-446.

Barker, D.C. and Tinnick, J.D., 2006. Competing visions of parental roles and ideological constraint. *American Political Science Review*, 100(2), pp.249-263.

Bennett, R., 1969. The intrinsic dimensionality of signal collections. *IEEE Transactions on Information Theory*, 15(5), pp.517-525.

Bobbio, N., 1996. Left and right: The significance of a political distinction. University of Chicago Press.

Carmines, E.G., Ensley, M.J. and Wagner, M.W., 2011. Issue preferences, civic engagement, and the transformation of American politics. *Facing the challenge of democracy: Explorations in the analysis of public opinion and political participation*, pp.329-354.

Cattell, R.B., 1966. The scree test for the number of factors. *Multivariate behavioral research*, 1(2), pp.245-276.

Chen, Feinian, Patrick J Curran, Kenneth A Bollen, James Kirby, and Pamela Paxton. 2008. "An empirical evaluation of the use of fixed cutoff points in rmsea test statistic in structural equation models". Sociological Methods & Research 36 (4): 462–494.

Claggett, W.J., Engle, P.J. and Shafer, B.E., 2014, July. The evolution of mass ideologies in modern American politics. In *The Forum* (Vol. 12, No. 2, pp. 223-256). De Gruyter.

Cochrane. Left and right: The small world of political ideas. McGill-Queen's Press-MQUP, 3, 2015.

Converse, P.E., 1964. The nature of belief systems in mass publics. Critical review, 18(1-3), pp.1-74.

Downs, Anthony. 1957. An Economic Theory of Democracy. New York: Harper and Row.

Duckitt, J. and Sibley, C.G., 2017. The dual process motivational model of ideology and prejudice. *The Cambridge handbook of the psychology of prejudice*, pp.188-221.

Ellis, C. and Stimson, J.A., 2012. *Ideology in America*. Cambridge University Press.

Eysenck, H.J., 1944. General social attitudes. Journal of Social Psychology, 19(2), p.207.

Feldman and Johnston. 2014 Understanding the determinants of political ideology: Implications of structural complexity. *Political Psychology*, 35(3):337–58.

Fiorina, M.P., Abrams, S.J. and Pope, J.C., 2011. Culture war?. Longman.

Free, Lloyd A., and Hadley Cantril. 1968. The Political Beliefs of Americans: A Study of Public Opinion. New York: Simon and Shuster.

Freeden, M., 2008. Thinking politically and thinking ideologically. *Journal of Political Ideologies 13(1)*, pp. 1-10.

Garrido, L.E., Abad, F.J. and Ponsoda, V., 2013. A new look at Horn's parallel analysis with ordinal variables. *Psychological methods*, *18*(4), p.454.

Golino, H., Moulder, R., Shi, D., Christensen, A.P., Garrido, L.E., Nieto, M.D., Nesselroade, J., Sadana, R., Thiyagarajan, J.A. and Boker, S.M., 2021. Entropy fit indices: New fit measures for assessing the structure and dimensionality of multiple latent variables. *Multivariate Behavioral Research*, 56(6), pp.874-902.

Hare, C., 2022. Constrained Citizens? Ideological Structure and Conflict Extension in the US Electorate, 1980–2016. *British Journal of Political Science*, *52*(4), pp.1602-1621.

Hochschild, J.L., 2001. Where you stand depends on what you see: Connections among values, perceptions of fact, and political prescriptions.

Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. Psychometrika, 30, 179 –185.

Hu, Li-tze and Peter M Bentler. 1999. "Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives". Structural Equation Modeling: a Multidisciplinary Journal 6 (1): 1–55.

Inglehart, Ronald. 1977. The Silent Revolution: Changing Values and Political Styles Among Western Publics. Princeton: Princeton University Press.

Inglehart, Ronald and Hans-Dieter Klingemann. 1976. "Party identification, ideological preference and the left- right dimension among western mass publics". In Party Identification and Beyond, pp. 243–273. Wiley London.

John, O. P., Donahue, E. M., & Kentle, R. L. (1991). The Big Five Inventory versions 4a and 54. Berkeley, CA: University of California, Berkeley, Institute of Personality and Social Research.

Jost JT, Federico CM, Napier JL: Political ideology: Its structure, functions, and elective affinities. Annu Rev Psychol 2009, 60:307-337.

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. Educational and Psychological Measurement, 20, 141–151. doi:10.1177/001316446002000116

Kalmoe, N.P., 2020. Uses and abuses of ideology in political psychology. *Political Psychology*, 41(4), pp.771-793.

Lauderdale BE, Hanretty C, Vivyan N: Decomposing public opinion variation into ideology, idiosyncrasy, and instability. J Politics 2018, 80:707-712.

Layman, G.C. and Carsey, T.M., 2002. Party polarization and "conflict extension" in the American electorate. *American Journal of Political Science*, pp.786-802.

Lentz, T.F., 1938. Generality and specificity of conservatism-radicalism. *journal of Educational Psychology*, 29(7), p.540.

Luo, S. (2005): unpublished study on personality traits and relationship satisfaction.

Lupton, R.N., Myers, W.M. and Thornton, J.R., 2015. Political sophistication and the dimensionality of elite and mass attitudes, 1980–2004. *The Journal of Politics*, 77(2), pp.368-380.

Marble, W. and Tyler, M., 2022. The Structure of Political Choices: Distinguishing Between Constraint and Multidimensionality. *Political Analysis*, *30*(3), pp.328-345.

Malka, A., Lelkes, Y. and Soto, C.J., 2019. Are cultural and economic conservatism positively correlated? A large-scale cross-national test. *British Journal of Political Science*, 49(3), pp.1045-1069.

Malka, A., Soto, C.J., Inzlicht, M. and Lelkes, Y., 2014. Do needs for security and certainty predict cultural and economic conservatism? A cross-national analysis. *Journal of personality and social psy-*

chology, 106(6), p.1031.

Maynard, J.L. and Mildenberger, M., 2018. Convergence and divergence in the study of ideology: A critical review. *British Journal of Political Science*, 48(2), pp.563-589.

Poole, K.T. and Rosenthal, H., 1985. A spatial model for legislative roll call analysis. *American journal of political science*, pp.357-384.

Rabinowitz, G. and Macdonald, S.E., 1989. A directional theory of issue voting. *American political science review*, 83(1), pp.93-121.

Reckase, M.D., 1990. Unidimensional Data from Multidimensional Tests and Multidimensional Data from Unidimensional Tests.

Rokeach, M., 1973. The nature of human values. Free press.

Schattschneider, Elmer Eric. 1960. *The Semisovereign People: A Realist's View of Democracy in America*. Holt, Rinehart and Winston.

Spearman, C., 1907. Demonstration of formulae for true measurement of correlation. *The American journal of psychology*, pp.161-169.

Sidanius, Jim; Pratto, Felicia,1999. *Social Dominance: An Intergroup Theory of Social Hierarchy and Oppression*. Cambridge: Cambridge University Press.

Stimson, J.A., 1975. Belief systems: Constraint, complexity, and the 1972 election. *American Journal of Political Science*, pp. 393-417.

Traag, V.A., Waltman, L. and Van Eck, N.J., 2019. From Louvain to Leiden: guaranteeing well-connected communities. *Scientific reports*, 9(1), p.5233.

Treier, S. and Hillygus, D.S., 2009. The nature of political ideology in the contemporary electorate. *Public Opinion Quarterly*, 73(4), pp.679-703.

Trunk, G.V., 1976. Stastical estimation of the intrinsic dimensionality of a noisy signal collection. *IEEE Transactions on Computers*, 100(2), pp.165-171.

Thurstone, L.L., 1934. The vectors of mind. Psychological review, 41(1), p.1.

Wilson, Glenn D.; Patterson, John R. (December 1968). "A New Measure of Conservatism". *The British Journal of Social and Clinical Psychology*. **7** (4): 264–269.

Tunguz, B. (2020) *Big five personality test*, *Kaggle*. Available at: https://www.kaggle.com/datasets/tunguz/big-five-personality-test (Accessed: 14 February 2024).

Wittgenstein. Philosophical investigations. Blackwell Publishers, 1953.

Zaller, J., 1992. The nature and origins of mass opinion. Cambridge university press.