Using Particle Physics to Model Non-linear Expressions of Latent Constructs

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Slides: <u>https://philip-warncke.netlify.app/files/PhaFNA_slides.pdf</u>

Social attitudes & latent constructs

Boss: This is the third time we've had complaints about your attitude this week, do you know what that means?

Me:





Research on social attitudes often involves **latent variables:**

- Intelligence
- Social trust
- Anxiety, depression, happiness
- Personality
- Political ideology
- ...







Data people provide







Assumptions of latent data-generating processes in LVM's

• Linearity

- Principle-Components analysis
- Maximum-likelihood factor models
- Monotonicity
 - Item-Response Theory
 - Weighted-least squares factor models
- <u>Linear constructs</u> (latent dimensions form cartesian space)
 - all models

In reality ...

- LDGP's may be
 - Non-linear
 - Non-monotonic

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We don't know the properties of the LDGP – need to infer these from manifest data!







Manifest data



Manifest data



Can latent variable models reproduce latent constructs when assumptions of linearity and/or dimensionality are violated?



-



Latent variable: Political ideology

Survey items:

- The United States should limit technology imports from China.
- The United States should increase military support to Ukraine.
- The people I disagree with politically are not evil.



Response curves — linear — exponential — plateau

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Latent variable: Political ideology

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Simulation setup

LDGP

- N = 1,000
- K = 12 items (5-point ordered Likert scales)
- 20 % measurement error

Latent variable models

- Principle components (PCA)
- Exploratory factor analysis (EFA)
- Item response theory (IRT)
- Physics-aided factornetwork analysis (PhaFNA)

Performance comparison

- Predicative accuracy (regression R-squared of predicted scores on latent construct)
- Computational time

Scenario 1: Ideal case





- Unidimensional
- Continuous
- Linear

LGDP

- 20% random measurement error
- Random-uniform transformation to 5-point ordered Likert-scale items



Manifest data

• 12 Likert scale items



Scenario 2: LGDP violates linearity



Latent variable

- Unidimensional
- Continuous
- Linear

LGDP

- 20% random measurement error
- Non-linear Likert transformations



- 12 Likert scale items
 - 2/12: Linear
 - 4/12: Quadratic
 - 4/12: 50% linear, 50% quadratic
 - 2/12: Exponential



Scenario 3: Latent construct violates linearity



Latent variable

- Continuous
- Non-linear

LGDP

- 20% random measurement error
- Linear and non-linear components fused together



Manifest data

- 12 5-point Likert scale items
- Items inherit 40% linear & 40% nonlinear component

Non-linear Latent Construct





t = 8.1s



Introducing PhaFNA

- Physics-aided, Factor Network Analysis
- Combines elements from factor analysis, IRT models, and statistical belief-network analysis
- Three basic ideas:
- 1. Treat item responses as physical particles that interact in latent space
- 2. Network communities of items physically anchor factors in latent space
- 3. Treats latent variables (i.e. factors) separate from latent space

Difference between latent factors and latent space

- Researchers commonly assume that latent factors live somewhere in a (wider) latent space
- Latent factors span the latent space in all conventional latent variable models
 - Assumptions about linearity and dimensionality **baked into** latent space
- PhaFNA models differ in that
 - Latent space and latent factors are conceptually separate
 - Latent space "just is" (predefined space)
 - Latent factors cut across the latent space in any possible direction

Latent space in PhaFNA



= Latent space

Latent space in PhaFNA



Latent space in PhaFNA



• **Spring-force**: positively correlated item-responses attract each other (increases with distance)



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- **Gravity:** more popular item responses are more massive (decreases with square of distance)







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- **Gravity:** more popular item responses are more massive (decreases with square of distance)
- **Drag:** All item-response particles lose momentum over time (ensures faster model convergence)








Demonstration

https://philip-warncke.netlify.app/files/demonstration.html



as.factor(dimension) 🛑 🔵

Non-linear Latent Construct



Applications

- Mapping the ideological space
- Tracing polarization across time

2000 ANES plot



2012 ANES plot

ANES

2012

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2020 ANES plot

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 V202344x 4
 V201342x - 3

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 V201258 6
 V201242x 6
 V202331x 3

 V202256 6
 V202248x 4
 V202344x 3
 V202248x 5
 V201349x 2

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V201350 3

ANES 2020

Mapping Polarization with Physics

Co-authored with Dino Carpentras (ETH Zurich), Yijing Chen (CEU), Bart de Bruin (Leiden), and Anne Speer (BIGSSS Bremen)



Conclusions

- Conventional latent variable models (LVM's) struggle when
 - LDGP's introduce non-linearities
 - Latent variables are non-linear
- PhaFNA is a novel latent variable modeling approach that simulates interactions between item responses in a physical latent space
- PhaFNA, more so than conventional LVM's is capable of
 - Restoring linear latent variables if LDGP's introduce non-linearities
 - Estimating non-linear latent variables

Next steps

- Developing diagnostic tools (non-linearities, multi-dimensionalities)
- Improving performance (e.g. implementation in Unity engine with GPU support)
- Implementation in higher dimensions

Please get in touch with collaboration ideas

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